STEADY-STATE ANALYSIS OF COUPLED-INDUCTOR ĆUK PWM CONVERTER
Part II: Discontinuous Conduction Mode

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Abstract: The paper presents a steady-state analysis of Ćuk PWM converter with the inductors coupled for discontinuous conduction mode (DCM). The results of this simplified analysis allow to determine the operating point of converter and further the small-signal low-frequency parameters of PWM switch model, and to design the converter too. The expressions of all steady-state currents and voltages take the effects of coupling into account. The converter with separate inductors is treated as a particular case of the more general coupled-inductor case.

Key-Words: Steady-state analysis, Coupled-inductor Ćuk PWM converter, discontinuous conduction mode

I. INTRODUCTION

For Ćuk PWM converter, a discontinuous conduction mode (DCM) means that the third time interval of operation cycle is nonzero, not that either inductor current is discontinuous (Ćuk 1978; Vorperian 1990). Coupled inductor techniques can be applied to Ćuk PWM converter with DCM to achieve ripple-free input or output current (Maksimovic et al. 1991; Sun et al. 2001).

The purpose of this paper is to provide a complete steady-state characterization for coupled-inductor Ćuk PWM converter with DCM. Taking the effects of coupling into account, the expressions of averaged input and output currents, and output voltage and voltage across energy storage capacitor are found for Ćuk converter with DCM, in Section II. The equation of the boundary between the two operating modes (CCM and DCM) is given in Section III.

The converter with separate inductors is treated as a particular case of the more general coupled-inductor case. Like as the analysis for CCM, the equations written on the circuit for each time interval corresponding to states of switches (transistor and diode) and the waveforms of electrical quantities have been used to obtain the mathematical model of the steady-state behavior of converter. We use the nomenclature given in (Niculescu 2003).

II. STEADY-STATE ANALYSIS OF ĆUK CONVERTER WITH DCM

Assume that the output current is now further decreased to enable the converter to enter into discontinuous conduction mode. The diagram of Ćuk PWM converter with the inductors coupled and DCM is given in Fig. 1. The waveforms of inductor currents and voltages corresponding to this operating mode are shown in Fig. 2.

Three distinct time intervals appears there, namely D_1T_s, D_2T_s and D_3T_s with $D_1+D_2+D_3=1$ for a constant switching frequency. For DCM, besides of expressions of averaged inductor currents, averaged input and output currents, averaged voltage across the energy storage capacitor C_1 and averaged currents transistor and diode, we have to determine the parameter D_2 that fixes the decay interval of inductor currents and the dc voltage conversion ratio implicitly.

Concerning the waveforms of inductor currents, it can see that the shapes of the currents in Fig. 2 are similar to those shown in Fig. 3 in (Niculescu 2003), the scales are different and the currents I_{L01} and I_{L02} are equal and have an inverse sense:

$$I_{L01}=-I_{L02}=I_{L0}. \tag{1}$$

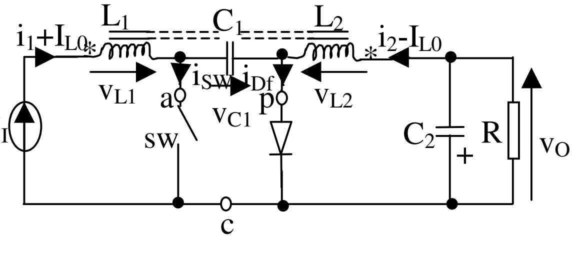
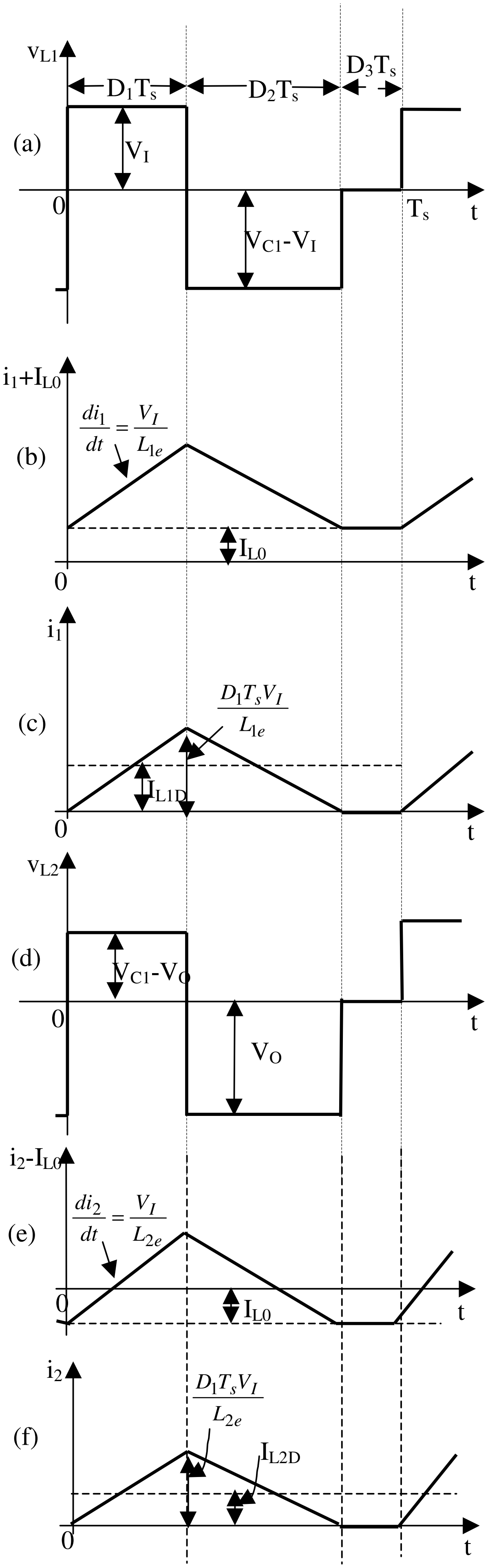


Fig.1. The diagram of Ćuk PWM converter with coupled inductors and DCM



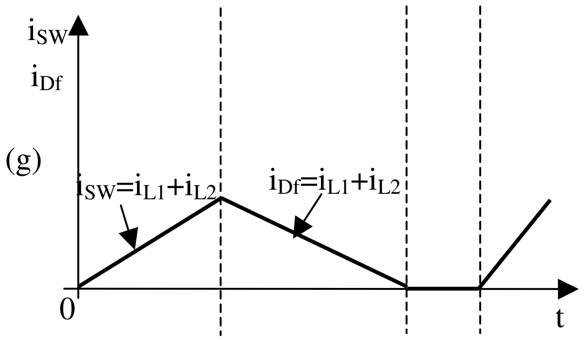


Fig.2. The waveforms of inductor currents and voltages for discontinuous conduction mode

For the first two time intervals D_1T_s and D_2T_s, the equation (9), (10) and (11), (12) in (Niculescu 2003) keep their validity and those will be repeated below, for an ease using. So, for $0 \le t \le D_1T_s$,

$$\frac{di_1}{dt} = \frac{V_I - \frac{k_c}{n}\left(V_{C1} - V_O\right)}{\left(1 - k_c^2\right)L_1} \tag{2}$$

$$\frac{di_2}{dt} = \frac{\left(V_{C1} - V_O\right) - k_c n V_I}{\left(1 - k_c^2\right)L_2}. \tag{3}$$

For $D_1T_s \le t \le \left(D_1 + D_2\right)T_s$,

$$\frac{di_1}{dt} = -\frac{\left(V_{C1} - V_O\right) - \frac{k_c}{n}V_O}{\left(1 - k_c^2\right)L_1} \tag{4}$$

$$\frac{di_2}{dt} = -\frac{V_O - k_c n\left(V_{C1} - V_O\right)}{\left(1 - k_c^2\right)L_2}. \tag{5}$$

For the third time interval (D_3T_s), the voltages across inductors are nulls: $v_{L1} = v_{L2} = 0$. Taking into account that the averaged voltage across the inductors over a switching period are null, the following relationships result:

$$V_{C1} = \left(1 + \frac{D_2}{D_1}\right)V_I = \left(1 + \frac{D_1}{D_2}\right)V_O. \tag{6}$$

The above relationships yield the dc voltage conversion ratio

$$M = \frac{V_O}{V_I} = \frac{D_1}{D_2} \tag{7}$$

and

$$V_{C1} = V_I + V_O. \tag{8}$$

The remark regarding the relationship (8) that is the same as (18) in (Niculescu 2003), made for the converter with CCM, keeps its validity for the converter with DCM. The determination of the conversion ratio M needs the value of parameter D_2 too. It is obvious that the conditions of ripple cancellation from the input or output current found for CCM remain unchanged for DCM.

The two effective inductances and two parameters of conduction through the inductors hold their expressions as in (Niculescu 2003), that is:

$$L_{1e} = \frac{\left(1-k_c^2\right)L_1}{1-k_c/n}\,;\; L_{2e} = \frac{\left(1-k_c^2\right)L_2}{1-k_c n}\,;$$

$$K_{1m} = \frac{2L_{1e}f_s}{R}\,;\; K_{2m} = \frac{2L_{2e}f_s}{R}\,.$$

Based on the waveforms of inductor currents, the formula of average value of ripple component of inductor currents can be written as:

$$I_{L1D} = \frac{D_1\left(D_1+D_2\right)T_sV_I}{2L_{1e}} = \frac{D_1\left(D_1+D_2\right)V_I}{K_{1m}R} \tag{9}$$

$$I_{L2D} = \frac{D_1\left(D_1+D_2\right)T_sV_I}{2L_{2e}} = \frac{D_1\left(D_1+D_2\right)V_I}{K_{2m}R}\,. \tag{10}$$

The calculation of these components of inductor currents needs to find the parameter D_2 firstly. In order to find the expression of the component I_{L0}, we use again the relationship of the converter efficiency, that is

$$V_O\left(I_{L2D}-I_{L0}\right) = \eta V_I\left(I_{L1D}+I_{L0}\right). \tag{11}$$

For 100% efficiency, the above equation yields

$$I_{L0} = \frac{V_OI_{L2D}-V_II_{L1D}}{V_O+V_I} \tag{12}$$

and further

$$I_{L0} = \frac{D_1V_I}{R}\left[\frac{D_1}{K_{1m}}-\frac{D_2}{K_{2m}}\right]. \tag{13}$$

As it can be seen from the waveforms of currents in the transistor and diode, shown as i_{SW} and respectively i_{Df} in Fig. 2 (g), the total rise of i_{SW} during $0 \le t \le D_1T_s$ should be equal to the total fall in i_{Df} during $D_1T_s \le t \le \left(D_1+D_2\right)T_s$. This means that we have

$$D_1T_s \times \left[\frac{di_1}{dt}+\frac{di_2}{dt}\right]_{0\le t\le D_1T_s} = D_2T_s \times \left[-\frac{di_1}{dt}-\frac{di_2}{dt}\right]_{D_1T_s\le t\le\left(D_1+D_2\right)T_s} \tag{14}$$

and after the substitution of the corresponding shapes of inductor current and using the relation (8), we find the dc voltage conversion ratio in (7) once more.

Now, we proceed to determine the formula for parameter D_2. For this purpose, we combine the equation (13) with

$$I_O = I_{L2} = I_{L2D}-I_{L0} = \frac{V_O}{R} = \frac{MV_I}{R}\,. \tag{15}$$

As results, the parameter D_2 is given by the formula

$$D_2 = \sqrt{K_{em}} \tag{16}$$

where the quantity

$$K_{em} = \frac{2L_{em}f_s}{R} \tag{17}$$

represents the parameter of conduction through an equivalent inductor with the inductance

$$L_{em} = L_{1e} \,//\, L_{2e}\,. \tag{18}$$

The averaged currents of transistor switch and diode result as follows:

$$I_{SW} = \frac{D_1}{D_1+D_2}\left(I_{L1D}+I_{L2D}\right) = \frac{D_1^2T_sV_I}{2L_{em}} = \frac{D_1^2V_I}{D_2^2R} = \frac{M^2V_I}{R} \tag{19}$$

$$I_{Df} = \frac{D_2}{D_1+D_2}\left(I_{L1D}+I_{L2D}\right) = \frac{D_1D_2T_sV_I}{2L_{em}} = \frac{D_1V_I}{D_2R} = \frac{MV_I}{R}\,. \tag{20}$$

III. BOUNDARY BETWEEN THE CONTINUOUS AND DISCONTINUOUS CONDUCTION MODES

For a complete characterization of the Ćuk PWM converter operating, we find now the equation of the boundary between CCM and DCM for the coupled-inductor case. It is well known that a converter will change the operating mode when the following equality is satisfied:

$$D_2 = 1-D_1\,. \tag{21}$$

This means that the parameter K_{em} reached its critical value

$$K_{emcrt} = \left(1-D_1\right)^2. \tag{22}$$

The same equation can be derived starting from the equality

$$I_{L01} = -I_{L02}\,. \tag{23}$$

The above equality defines the condition of passing from CCM to DCM too, if there the parameter K_{em} will have its critical value K_{emcrt}. Also, all these results characterize the Ćuk PWM converter with separate inductors, case in which we have to set k_c at zero.

IV. CONCLUSION

Formally, the dc conversion ratio of Ćuk PWM converter with coupled-inductors and operating in DCM has the same formula as in the separate-inductor case. But, the parameter D_2 takes the coupling effect into account, by means of the parameter of conduction through an equivalent inductor with the inductance L_{em}. The averaged input and output currents, and the averaged currents of transistor and diode as the boundary equation include the coupling effect too. So, the steady-state operating point and the parameters of small-signal low-frequency model of converter depend on the coupling parameter. Moreover, the steady-state mathematical model developed here supplies the formula of equivalent inductance in the input and output circuits of converter that are entailed by a dynamic analysis of converter based on its equivalent circuit.

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